Tables of Generating Functions, Reduced and Representative for certain Ternary Systems of Binary Forms.

By J. J. Sylvester.

The annexed tables have been calculated under my directions by Messrs. Durfee and Ely, out of the fund placed at my disposition by the British Association for the Advancement of Science in the year 1881. Subsequent investigation will be necessary in order to ascertain whether there exist or not extra tabular groundforms which escape the operation of tamisage.

G. F. it will be understood stands for the words Generating Function.

System of Two Quadratics and One Quartic.

G. F. for invariants, reduced form.

Denominator: $(1 - b^2)(1 - \beta^2)(1 - d^2)(1 - d^3)(1 - b\beta)(1 - bd)$
$(1 - \beta d)(1 - b^2 d)(1 - \beta^2 d)$.

Numerator:

		d^0	d^1	d^2	d^3	d^4
b^0	β^0	1				
	β^1		$\bar{1}$			
	β^2			1		
b^1	β^0		$\bar{1}$			
	β^1		1	2		
	β^2		1		$\bar{1}$	
	β^3			$\bar{1}$		

		d^0	d^1	d^2	d^3	d^4
b^3	β^1			$\bar{1}$		
	β^2				1	
	β^3					$\bar{1}$
b^2	β^0			1		
	β^1		1	$\bar{1}$		
	β^2			$\bar{2}$	$\bar{1}$	
	β^3			1		

G. F. for invariants, representative form.

Denominator: $(1 - b^2)(1 - \beta^2)(1 - d^2)(1 - d^3)(1 - b\beta)(1 - b^2 d^2)$
$(1 - \beta^2 d^2)(1 - b^2 d)(1 - \beta^2 d)$.

Numerator:

		d^0	d^1	d^2	d^3	d^4	d^5
	β^0	1					
b^0	β^1						
	β^2						
	β^3				1		
	β^4						
	β^0						
	β^1		1	1			
b^1	β^2		1	1	1		
	β^3						
	β^4				$\bar{1}$		
	β^0						
b^2	β^1		1	1	1		
	β^2						
	β^3				$\bar{1}$	$\bar{1}$	$\bar{1}$
	β^4						

		d^0	d^1	d^2	d^3	d^4	d^5	d^6
	β^0							
b^4	β^1				$\bar{1}$			
	β^2							
	β^3							
	β^4							$\bar{1}$
	β^0				1			
	β^1							
b^3	β^2				$\bar{1}$	$\bar{1}$	$\bar{1}$	
	β^3					$\bar{1}$	$\bar{1}$	
	β^4							

TABLE OF GROUNDFORMS.

Deg. in coeff's of quadratic	Deg. in coeff's of quadratic	Deg. in coeff's of quartic			
		0	1	2	3
	0			1	1
0	1				
	2	1	1	1	
	3				1
	0				
1	1	1	1	1	
	2		1	1	1
2	0	1	1	1	
	1		1	1	1
3	0				1

SYSTEM OF QUADRATIC, CUBIC, AND QUARTIC.

G. F. for invariants, reduced form.

Denominator: $(1 - b^2)(1 - c^4)(1 - d^2)(1 - d^3)(1 - bc^2)(1 - b^3c^2)$
$(1 - bd)(1 - b^2d)(1 - c^2d)(1 - c^2d^3)(1 - c^4d)$
$(1 - c^4d^3).$

Numerator :

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}
b^0	c^0	1										
	c^2		$\overline{1}$									
	c^4			2	2	2	1					
	c^6			1	1		$\overline{1}$	$\overline{1}$				
	c^8				$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{2}$				
	c^{10}								1			
	c^{12}									$\overline{1}$		
b^1	d^0		$\overline{1}$									
	c^2		2	4	2	1						
	c^4		2	2		$\overline{1}$	$\overline{2}$	$\overline{1}$				
	c^6			$\overline{2}$	3	3	$\overline{1}$	1	1			
	c^8			$\overline{1}$	$\overline{1}$		1	1	1			
	c^{10}				1	1		$\overline{1}$	$\overline{2}$	$\overline{2}$		
	c^{12}							$\overline{1}$		1		
	c^{14}								1			
b^2	c^0			1								
	c^2		2	1	$\overline{1}$	$\overline{1}$	$\overline{1}$					
	c^4		1	$\overline{1}$	$\overline{1}$	$\overline{1}$	$\overline{1}$					
	c^6			$\overline{1}$			1	1				
	c^8			$\overline{1}$	$\overline{1}$	2	$\overline{2}$	$\overline{1}$				
	c^{10}						$\overline{1}$	$\overline{1}$		1	1	
	c^{12}						1	2	2	1	2	1
	c^{14}										$\overline{1}$	
b^3	c^2		1	$\overline{1}$	$\overline{1}$	$\overline{1}$						
	c^4	1	1		1			$\overline{1}$				
	c^6		$\overline{2}$	$\overline{3}$	$\overline{2}$	$\overline{2}$	1	$\overline{2}$	1	$\overline{1}$		
	c^8		$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{1}$		1	2	1	1	
	c^{10}			1	1	2	1	2	2	3	2	
	c^{12}					1			$\overline{1}$		$\overline{1}$	$\overline{1}$
	c^{14}							1	1	1	$\overline{1}$	

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}
b^6	c^4			1								
	c^6				$\overline{1}$							
	c^8					2	2	2	1			
	c^{10}					1	1		$\overline{1}$	$\overline{1}$		
	c^{12}						$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{2}$		
	c^{14}										1	
	c^{16}											$\overline{1}$
b^5	c^2			$\overline{1}$								
	c^4		$\overline{1}$		1							
	c^6			2	2	1		$\overline{1}$	$\overline{1}$			
	c^8				$\overline{1}$	$\overline{1}$	$\overline{1}$		1	1		
	c^{10}				$\overline{1}$	1	1	3	3	2		
	c^{12}					1	2	1		$\overline{2}$	$\overline{2}$	
	c^{14}							$\overline{1}$	2	$\overline{4}$	$\overline{2}$	
	c^{16}									1		
b^4	c^2		1									
	c^4		$\overline{1}$	2	1	2	2	1				
	c^6		$\overline{1}$	$\overline{1}$			1	1				
	c^8					1	2	2	1	1		
	c^{10}					$\overline{1}$	$\overline{1}$		1			
	c^{12}						1	1	1	1	$\overline{1}$	
	c^{14}						1	1	1	$\overline{1}$	$\overline{2}$	
	c^{16}									$\overline{1}$		

G. F. for invariants, representative form.

Denominator: $(1-b^2)(1-c^4)(1-d^2)(1-d^3)(1-bc^2)(1-b^3c^2)(1-b^2d^2)$
$(1-b^2d)(1-c^4d^2)(1-c^2d^3)(1-c^4d)(1-c^4d^3)$

Numerator:

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}	d^{11}
b^0	c^0	1											
	c^2												
	c^4			1	2	2	1						
	c^6			1	3	2	1						
	c^8					$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{1}$				
	c^{10}					$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{1}$				
	c^{12}												
	c^{14}							$\overline{1}$					
b^1	c^2		2	3	2	1							
	c^4		2	4	5	3	1						
	c^6												
	c^8			$\overline{1}$	$\overline{3}$	$\overline{3}$	3	2	1				
	c^{10}					$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{2}$				
	c^{12}				1	1		$\overline{2}$	$\overline{2}$	$\overline{1}$			
	c^{14}												
	c^{16}								1				
b^2	c^2		2	3	3	1							
	c^4		1	3	4	2	$\overline{1}$	$\overline{2}$	$\overline{1}$				
	c^6				1	2	3	2	1				
	c^8			$\overline{1}$	$\overline{3}$	$\overline{5}$	$\overline{5}$	$\overline{2}$	1	2	1		
	c^{10}					$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{2}$	$\overline{1}$			
	c^{12}						1	2	2	1			
	c^{14}						1	2	2	1	1	1	1
b^3	c^0			1									
	c^2		1	1		$\overline{1}$	$\overline{1}$	$\overline{1}$					
	c^4	1	1	2	1	$\overline{1}$	$\overline{3}$	$\overline{3}$	$\overline{1}$				
	c^6		$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{1}$			
	c^8		$\overline{1}$	$\overline{3}$	$\overline{6}$	$\overline{5}$	$\overline{4}$	$\overline{2}$		1			
	c^{10}				$\overline{1}$	$\overline{2}$	$\overline{1}$	1	1	4	4	2	
	c^{12}			1	2	3	2	2	3	4	3	1	
	c^{14}					1	2	3	2				
	c^{16}							1	1	1	$\overline{1}$	$\overline{1}$	

		d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}	d^{11}	d^{12}
b^7	c^4			1									
	c^6												
	c^8				1	2	2	1					
	c^{10}				1	3	2	1					
	c^{12}						$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{1}$			
	c^{14}						$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{1}$			
	c^{16}												
	c^{18}												$\overline{1}$
b^6	c^2			$\overline{1}$									
	c^4												
	c^6			1	2	2	$\overline{1}$	$\overline{1}$					
	c^8				2	3	2	1					
	c^{10}					1	2	3	3	3	1		
	c^{12}												
	c^{14}							$\overline{1}$	$\overline{3}$	$\overline{5}$	$\overline{4}$	$\overline{2}$	
	c^{16}							$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{2}$		
b^5	c^4	$\overline{1}$	$\overline{1}$	$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{1}$					
	c^6				$\overline{1}$	$\overline{2}$	$\overline{2}$	$\overline{1}$					
	c^8					1	2	3	2	1			
	c^{10}				$\overline{1}$	$\overline{2}$	$\overline{1}$	2	5	5	3	1	
	c^{12}						1	2	3	2	1		
	c^{14}						1	2	1	$\overline{2}$	$\overline{4}$	$\overline{3}$	$\overline{1}$
	c^{16}								$\overline{1}$	$\overline{3}$	$\overline{3}$	$\overline{2}$	
b^4	c^2	1	1	$\overline{1}$	$\overline{1}$	$\overline{1}$							
	c^4			$\overline{2}$	$\overline{3}$	$\overline{2}$	$\overline{1}$						
	c^6	$\overline{1}$	$\overline{3}$	$\overline{4}$	$\overline{3}$	$\overline{2}$	2	3	2	1			
	c^8		$\overline{2}$	$\overline{4}$	$\overline{4}$	1	1	2	1				
	c^{10}				$\overline{1}$	2	4	5	6	3	1		
	c^{12}					1	2	2	2	2	2	1	
	c^{14}					1	3	3	1	$\overline{1}$	$\overline{2}$	$\overline{1}$	1
	c^{16}							1	1	1	$\overline{1}$	$\overline{1}$	
	c^{18}									$\overline{1}$			

TABLE OF GROUNDFORMS.

Deg. in coeff's of Quadratic	Deg. in coeff's of Cubic.	Degree in coeff's of Quartic.					
		0	1	2	3	4	5
0	0			1	1		
	2				1		
	4	1	1	2	3	2	1
	6			1	3	2	1
1	2	1	2	3	2	1	
	4		2	4	5	3	1
2	0	1	1	1			
	2		2	3	3	1	
	4	1					
3	0				1		
	2	1	1	1			
	4	1	1				
4	2		1	1			

SYSTEM OF ONE QUADRATIC AND TWO QUARTICS.

G. F. for invariants, reduced form.

Denominator: $(1-b^2)(1-\delta^2)(1-\delta^3)(1-d^2)(1-d^3)(1-b\delta)(1-b^2\delta)$
$(1-bd)(1-b^2d)(1-\delta d)(1-\delta^2 d)(1-\delta d^2)$

Numerator :

		d^0	d^1	d^2	d^3	d^4	d^5	d^6
b^0	δ^0	1						
	δ^1							
	δ^2			1				
	δ^3							
	δ^4				1			
b^1	δ^0		$\overline{1}$					
	δ^1	$\overline{1}$	1	1	1			
	δ^2		1	1				
	δ^3		1		1			
	δ^4					$\overline{1}$		
	δ^5				$\overline{1}$			
b^2	δ^0			1				
	δ^1		2		$\overline{1}$	$\overline{1}$		
	δ^2	1		$\overline{1}$	$\overline{2}$			
	δ^3		$\overline{1}$	$\overline{2}$				
	δ^4		$\overline{1}$			$\overline{1}$		
	δ^5				$\overline{1}$	1		

		d^0	d^1	d^2	d^3	d^4	d^5	d^6
b^5	δ^2			1				
	δ^3							
	δ^4					1		
	δ^5							
	δ^6							1
b^4	δ^1			$\overline{1}$				
	δ^2		$\overline{1}$					
	δ^3				1		1	
	δ^4					1	1	
	δ^5				1	1	1	$\overline{1}$
	δ^6						$\overline{1}$	
b^3	δ^1		1	$\overline{1}$				
	δ^2		$\overline{1}$				$\overline{1}$	
	δ^3					$\overline{2}$	$\overline{1}$	
	δ^4				$\overline{2}$	1		1
	δ^5			$\overline{1}$	$\overline{1}$		2	
	δ^6					1		

G. F. for invariants, representative form.

Denominator: $(1 - b^2)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3)(1 - b^2\delta^2)(1 - b^2\delta)$
$(1 - b^2 d^2)(1 - b^2 d)(1 - \delta d)(1 - \delta^2 d)(1 - d^2\delta).$

Numerator:

		d^0	d^1	d^2	d^3	d^4	d^5	d^6
	δ^0	1						
	δ^1							
b^0	δ^2			1				
	δ^3							
	δ^4					1		
	δ^1		1	1	1			
b^1	δ^2		1	1	1			
	δ^3		1	1	1			
	δ^1		1	1				
	δ^2		1	1				
b^2	δ^3				1	1		
	δ^4				1		$\bar{1}$	$\bar{1}$
	δ^5					$\bar{1}$		
	δ^6					$\bar{1}$		
	δ^0				1			
	δ^1		1	1		$\bar{1}$	$\bar{1}$	
b^3	δ^2		1	1	$\bar{1}$	$\bar{2}$	$\bar{1}$	
	δ^3	1		$\bar{1}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	
	δ^4		$\bar{1}$	$\bar{2}$	$\bar{2}$			
	δ^5		$\bar{1}$	$\bar{1}$	$\bar{1}$			

		d^1	d^2	d^3	d^4	d^5	d^6	d^7
	δ^3			1				
	δ^4							
b^7	δ^5					1		
	δ^6							
	δ^7							1
	δ^4				1	1	1	
b^6	δ^5				1	1	1	
	δ^6				1	1	1	
	δ^1			$\bar{1}$				
	δ^2			$\bar{1}$				
b^5	δ^3	$\bar{1}$	$\bar{1}$		1			
	δ^4			1	1			
	δ^5					1	1	
	δ^6					1	1	
	δ^2				$\bar{1}$	$\bar{1}$	$\bar{1}$	
	δ^3				$\bar{2}$	$\bar{2}$	$\bar{1}$	
b^4	δ^4		$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{1}$		1
	δ^5		$\bar{1}$	$\bar{2}$	$\bar{1}$	1	1	
	δ^6		$\bar{1}$	$\bar{1}$		1	1	
	δ^7				1			

TABLE OF GROUNDFORMS.

Deg. in coeff's of quadratic.	Deg. in coeff's of quartic.	Deg. in coeff's of quartic.			
		0	1	2	3
0	0			1	1
0	1		1	1	
0	2	1	1	1	
0	3	1			
1	0				
1	1		1	1	1
1	2		1	1	1
1	3		1	1	
2	0	1	1	1	
2	1	1	1	1	
2	2	1	1		
3	0				1
3	1		1	1	
3	2		1		
3	3	1			

SYSTEM OF THREE QUARTICS.

G. F. for invariants, reduced form.

Denominator: $(1 - \partial^2)(1 - \partial^3)(1 - \delta^2)(1 - \delta^3)(1 - d^2)(1 - d^3)$
$(1 - \partial\delta)(1 - \partial d)(1 - \delta d)(1 - \partial^2 d)(1 - \partial d^2)$
$(1 - \partial^2 \delta)(1 - \partial\delta^2)(1 - \delta^2 d)(1 - \delta d^2).$

Numerator:

Left half:

∂	δ	d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8
∂^0	δ^0	1								
	δ^1									
	δ^2			1						
	δ^3									
	δ^4					1				
∂^1	δ^1		1	1						
	δ^2		1	1						
	δ^3				1	1				
	δ^4				1		$\bar{1}$	$\bar{1}$		
	δ^5				$\bar{1}$					
	δ^6				$\bar{1}$					
∂^2	δ^0			1						
	δ^1		1	1						
	δ^2	1	1	1			$\bar{1}$	$\bar{1}$		
	δ^3				1	$\bar{1}$	2	$\bar{1}$		
	δ^4				$\bar{1}$	$\bar{3}$	$\bar{2}$		1	
	δ^5			$\bar{1}$	$\bar{2}$	$\bar{2}$	1	1		
	δ^6			$\bar{1}$	$\bar{1}$		1	1		
	δ^7					1				
∂^3	δ^1				1	1				
	δ^2				1	$\bar{1}$	$\bar{2}$	$\bar{1}$		
	δ^3		1	1		$\bar{4}$	$\bar{3}$	$\bar{1}$		
	δ^4		1	$\bar{1}$	$\bar{4}$	$\bar{5}$		2	1	
	δ^5			$\bar{2}$	$\bar{3}$		3	2		
	δ^6			$\bar{1}$	$\bar{1}$	2	2	1		
	δ^7					1				

Right half:

∂	δ	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8
∂^8	δ^4				$\bar{1}$				
	δ^5								
	δ^6						$\bar{1}$		
	δ^7								
	δ^8								$\bar{1}$
∂^7	δ^2				1				
	δ^3				1				
	δ^4		1	1	$\bar{1}$				
	δ^5				$\bar{1}$	$\bar{1}$			
	δ^6						$\bar{1}$	$\bar{1}$	
	δ^7						$\bar{1}$	$\bar{1}$	
∂^6	δ^1				$\bar{1}$				
	δ^2		$\bar{1}$	$\bar{1}$		1	1		
	δ^3		$\bar{1}$	$\bar{1}$	2	2	1		
	δ^4	$\bar{1}$		2	3	1			
	δ^5		1	2	1	$\bar{1}$			
	δ^6		1	1			$\bar{1}$	$\bar{1}$	$\bar{1}$
	δ^7						$\bar{1}$	$\bar{1}$	
	δ^8						$\bar{1}$		
∂^5	δ^1				$\bar{1}$				
	δ^2		$\bar{1}$	2	$\bar{2}$	1	1		
	δ^3		$\bar{2}$	$\bar{3}$		3	2		
	δ^4	$\bar{1}$	$\bar{2}$		5	4	1	$\bar{1}$	
	δ^5		1	3	$\bar{4}$		$\bar{1}$	$\bar{1}$	
	δ^6		1	2	1	$\bar{1}$			
	δ^7				$\bar{1}$	$\bar{1}$			

Numerator— *Continued :*

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8
	δ^0					1				
	δ^1					1	$\overline{1}$	$\overline{1}$		
	δ^2				$\overline{1}$	$\overline{3}$	$\overline{2}$		1	
∂^4	δ^3		1	$\overline{1}$	$\overline{4}$	$\overline{5}$		2	1	
	δ^4	1		$\overline{3}$	$\overline{5}$		5	3		$\overline{1}$
	δ^5		$\overline{1}$	$\overline{2}$		5	4	1	$\overline{1}$	
	δ^6		$\overline{1}$		2	3	1			
	δ^7			1	1		$\overline{1}$			
	δ^8					$\overline{1}$				

Representative form same as reduced form.

TABLE OF GROUNDFORMS.

Deg. in coeff's of quartic	Deg. in coeff's of quartic	Deg. in coeff's of quartic			
		0	1	2	3
0	0			1	1
	1		1	1	
	2	1	1	1	
	3	1			
1	0		1	1	
	1	1	1	1	
	2	1	1	1	
2	0	1	1	1	
	1	1	1	1	
	2	1	1		
3	0	1			